

Combinatorics
Semestral Examination
8th May 2013

Instructions: All questions carry equal marks.

1. Define *circulation* on a diagraph D . Prove that if f is a circulation on a finite diagraph D , then there exists an integral circulation g whose value for all edges e equals either $\lfloor f(e) \rfloor$ or $\lceil f(e) \rceil$.
2. Define *Steiner triple system*. Prove that a necessary condition for a Steiner triple system to exist on v points is $v \equiv 1$ or $3 \pmod{6}$. Prove that for any number $v = 6t + 3$, a Steiner triple system on v points exists.
3. Define a combinatorial geometry. Explain what one means by the closure of a set in a combinatorial geometry and state and prove the *exchange axiom* of combinatorial geometry.
4. Let E and F be flats in a combinatorial geometry. State and prove the semi-modular law of combinatorial geometries
5. Define *projective geometry*. Prove that the induced subgeometry on any flat F of a projective geometry is also a projective geometry.